It's a Coyote Eat Deer Feed Tick World: A Deterministic Model of Predator-Prey Interaction in the Northeast

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Abstract

Occurrences of Lyme Disease have dramatically increased since the disease was named in the 1970's. Much research now focuses on controlling Lyme through ticks (the vector for the disease), or White Tailed Deer (the host for ticks). Deer have recently reached surprisingly high numbers in the Northeast and it is thought that reducing deer populations will effectively decrease tick populations and thus the threat of Lyme. Consequently many towns have considered implementing ambitious yet controversial deer culling programs. As an alternative, we look at the potential for coyotes to biologically control deer populations through predation.

Coyotes, who prey on deer, have recently migrated into the Northeast from the Plains and may have been attracted to their new territory by the abundant prey supply. It is questioned whether the coyotes will act to replace the wolf as a natural control on deer population, thereby reducing Lyme Disease.

We construct a deterministic model to represent the current deer and coyote population dynamics and use this model to investigate the long term interaction of coyotes and deer. We explore the potential for coyotes to either solely act as a biological control on the deer populations or aid deer culling programs. The model is explored analytically and numerically and predicts that significant human intervention is needed to successfully control deer. Thus numerical simulations of the model and possible culling programs are provided to help highlight the system dynamics and guide culling policies.

Introduction

Recently the northeastern region of the United States has suffered an explosion of both White-Tailed Deer and cases of Lyme Disease. These two explosions are not considered to be independent and both issues greatly concern residents and policy makers of the area.

It is thought the deer explosion is a result of an increase in human population density. As humans developed the area, they created more disturbed landscapes that favor the growth of grasses, a major deer food source. Human development also conflicted with large carnivores and resulted in humans effectively killing wolves and natural ungulate predators in the region. Without predation, deer populations burgeoned in the last century [5]. Populations have gotten so large that deer have been reported in some areas at population densities as high as ten times what is thought to be healthy deer densities (20 deer per square mile vs. 200 deer per square mile) [5, 22]. Having such dense deer populations is dangerous; it leads to increased deer-car collisions [11, 24], over-grazing that can destroy natural

forests, and endangerment of indigenous species [15, 4]. Possibly the greatest danger of a dense deer population, (though), is that it increases the risk of contracting Lyme disease [11, 26, 25, 22].

Lyme Disease is caused by at least three spirochetal bacteria in the genus Borrealia, but most commonly by Borrealia burgforferi. The spirochete is transmitted to individuals by ticks of the genus Ixodes. When a tick bites a host to feed on their blood, the tick transmits the spirochete to the host through its saliva and thus infects the host. Because adult Ixodes ticks frequently feed on the blood of White Tailed Deer, it is thought that more deer mean more tick and thus more Lyme. Municipalities have thus begun to explore the possibility of protecting their citizens from Lyme disease and dangers of deer overpopulation with deer control programs [5]. These programs usually propose periodically culling a portion of the deer population. Such programs have many drawbacks.

Culling is expensive and diffiult to practice as annual surveys are needed to count and monitor deer populations. Further difficulties arise with residents as the majority of land is private-

ly owned and not all citizens morally agree with killing deer or bringing the dangers of hunting close to their homes.

We want to look for another way to potentially control deer without having to cull seasonally. We look for a biological control in the area that could act to control deer in adequate time. Specifically, we investigate if the effect of the emergent Eastern Coyote would be enough to act as this biological control.

In addition to allowing explosive deer growth, the extinction of wolves in the region has opened a niche for larger carnivores. As the emergence of a resident coyote population has occurred simultaneously, some suggest the coyote, which preys on ungulates, may be filling the wolf niche [14, 2]. Coyotes indigenous to the Great Plains and Southwest United States began migrating east and have successfully established populations in areas as far as New Brunswick and Nova Scotia [17, 2]. The eastern migration of the coyote has happened rather quickly and is thought to have resulted in a population of coyotes that is drastically different from the western coyote. The eastern coyote is physically larger [23, 13, 18, 8, 12] and has a larger home range than its western counterpart [27]. It is unknown why coyotes have moved so quickly to the Northeast, but it is believed that coyotes were attracted by abundant prey [23].

Very little is known about New England's newest resident carnivore. It is necessary to learn about coyotes in the Northeast, as their new habitats of suburbia and New England deciduous forest differ greatly from their original home of expansive plains or the arid Southwest and puts coyotes in close proximity to humans. However, mammals with large home ranges are incredibly difficult to track and study, so other methods of study are needed to investigate and understand coyotes on a broad scale in the Northeast [9]. These studies are especially needed if coyotes present a possible predator for deer and municipalities are looking for a method to control deer. Currently there seems to be no literature that looks at potential impact from the growth of the coyote population on Lyme. Further it appears no research has looked at exploiting coyotes as a new natural predator and control on deer populations.

We hope that employing a mathematical model might be a good way to study coyote dynamics and overcome the problems of tracking coyotes. We hope to gain insight into whether coyotes will have a significant impact on bringing deer down to acceptable densities and if the coyotes can achieve this feat within some reasonable time frame. As humans have a low tolerance of coyotes, we are also interested in if the coyotes can accomplish deer reduction with low coyote densities. Since we find mathematically that coyotes alone will not successfully control deer, we want to investigate the different types, impacts, and efficiencies of culling programs that towns might pursue. We hope to find a program that will allow us to suggest a minimally invasive cull to minimize expenses by only culling the minimum number of deer.

Modeling

Preliminary Assumptions and Notations

For simplicity, we ignore spatial variation and focus on a one square mile spatially homogeneous closed system. While this prevents modeling the varied distribution of fauna across the landscape, it will allow us to make estimates for larger scales such as multi-state domains. We also use similar assumptions to average predation and growth over a year. There is naturally some annual variability in growth and predation as birth rates and deer vulnerability vary on seasonal conditions such as snow depth [16, 17]. However this assumption is necessary to maintain an autonomous system of equations for analysis; it will also not inhibit our goal of studying the long term population dynamics.

We wish to model the density of deer D(t) and coyote C(t) populations with respect to time t in months. We begin with a classical Lotka-Volterra predator prey system,

$$\frac{dD}{dt} = rD - aDC,$$

$$\frac{dC}{dt} = eaDC - dC,$$

where r is deer growth rate, a is the proportion of deer \that die in coyote-deer interactions, e is the energy that coyotes get from each killed deer, and d is the death rate for coyotes. Note this model is too simplistic, as in the absence of coyote predation, this model assumes exponential growth for deer and, in the
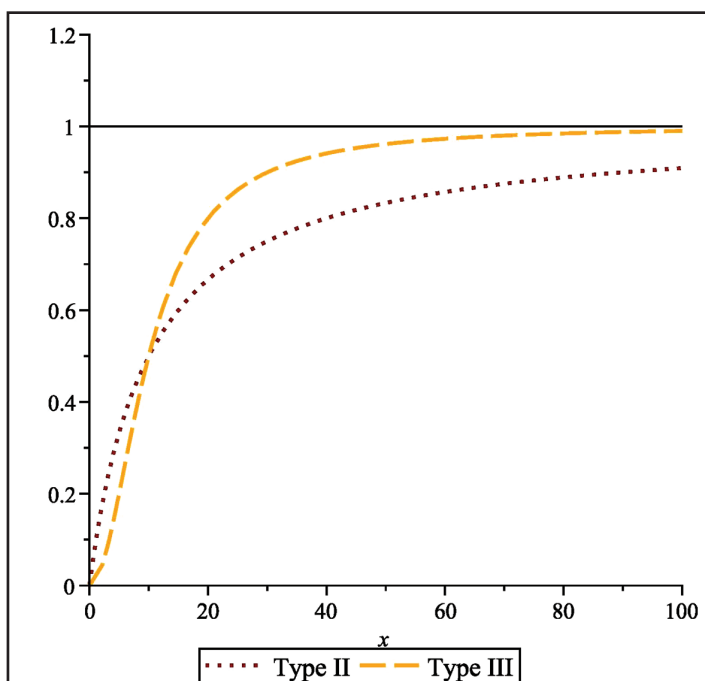

Figure 1: Figure 1: Holling type II and III predation terms model the number of deer consumed by one coyote in one month dependent on deer density. Both terms are constrained by =1, the chosen maximum deer a coyote can consume per month. (Graphic courtesy of Kathy Li)

absence of deer prey, exponential decay for coyotes. Therefore we introduce terms to more realistically portray coyote and deer interaction and reliance on the environment.

Predation

As in the classical model, we assume that at reasonable densities, deer die only as a result of coyote-deer interactions. In our sample area, this assumption is nearly reasonable as other natural predators of deer (wolves and large cats) are virtually extinct in the area. To represent this density-dependent predation, we use a Holling type II functional response term [10].

Classical mass-action predation terms such as aDC show that as prey increase, the number of prey killed by each predator increases. This is accurate if prey populations are relatively small. If prey become dense, mass action says that each coyote will kill proportionally large numbers of deer. It is more realistic that each coyote will have a maximum number of deer it can eat each month, regardless of how gigantic a population may be. Whether each coyote's predation is maximized depends on if there are ample deer.

Holling type terms allow us to model such a cap for individual coyote predation (see Figure 1). A Holling type III term is frequently used to model mammal predation as it shows prey switching and lower predation rates if primary prey abundance is low [1]. However as studies carried out by Patterson (2000) indicate, the coyote-deer interactions better resembles a type II response

$$\frac{\alpha DC}{\beta + D}$$

where α is the maximum deer that a single coyote will consume in one month and β controls how quickly the predation reaches α.

Note that a problem with Holling predation is that it does assume deer must reach an infinite population before individual coyotes maximize their prey consumption. Finally, we modify the Holling term to take into account the density of predators, to include C in coyote-deer interaction term.

Prey Growth

To model deer growth, we choose to use a logistic term:

$$R_d D(1 - \frac{D}{K_d}),$$

where R_d is a natural growth rate for the deer population and K_d is a carrying capacity for deer. (The issue of how to chose the carrying capacity is extremely difficult, important to both coyotes and deer, and discussed later.) While the deer populations have exploded in a relatively short time, it does not seem reasonable to assume that deer continually reproduce exponentially. Furthermore, if we were to assume exponential growth and

run our model with initial values as large as current population estimates, then the D(t) would quickly overrun the model and inhibit any sort of reasonable study. Recent field surveys also show that deer populations seem to be stabilizing, though only at extremely high densities [7].

Predator Growth

We want the coyote population to be correlated with deer and to grow with increased deer, which the Holling term allows. However if we use the Holling term with the extremely high populations of deer that have been observed in the Northeast, then the model will predict exploding coyote population. As there are reports of human-coyote encounters and coyotes killing pets, coyotes are often seen as a threat by humans and it is reasonable to assume that humans will control coyotes [3, 27]. If the coyote population gets too large, humans will begin killing coyotes. It is reasonable to assume that the density of coyotes that humans will tolerate is below the density of the coyotes that the current prey population could support. To impose this anthropogenic growth barrier, we multiply predation by a logistic growth term with carrying capacity Kc. The importance of Kc will become apparent later.

$$\frac{\gamma \alpha DC}{\beta + D}(1 - \frac{C}{K_c})$$

We also want to consider that coyotes prey on a variety of forest creatures and fruit: they are not solely dependent on the presence of deer [21, 16]. Hence we introduced a term for coyote growth independent of deer:

$$R_c C(1 - \frac{C}{K_c})$$

This term allows R_c to represent the growth from coyotes preying on species other than deer. Multiplying by a logistic term $(1 - K_c)$ represents the human barrier imposed on this term too. Then in the absence of deer, the coyote population will grow extremely slowly.

Carrying Capacity

The issue of carrying capacity is a delicate issue that arises quite frequently in biological problems. Mathematically, the carrying capacity represents a value which, if the density exceeds this value, growth becomes negative and the population decays back to the carrying capacity. Biologically, carrying capacity is the maximum density of a species that a given habitat can support long-term in ideal conditions.

Mathematically, carrying capacities are typically constants because they approximate long term dynamics. However, many factors affect biological carrying capacity such as food supply, climate, and over crowding. Carrying capacities can vary with

respect to what is considered an external factor, such as human development, or an internal factor, such as overabundant inhabitants overgrazing and damaging the environment, thus diminishing their food source. Due to different factors, carrying capacities are an extremely delicate matter; we take Kd, Kc to be constant within our model, however, and find reasonable values from literature that predict maximum population levels.

Specialized Coyote-Deer Model

Taking into consideration all of the terms developed above, we end up with the following model:

$$(a) \quad \frac{dD}{dt} = R_d D(1 - \frac{D}{K_d}) - \frac{\alpha DC}{\beta + D}$$

$$(b) \quad \frac{dC}{dt} = (1 - \frac{C}{K_c})(R_c C + \frac{\gamma \alpha DC}{\beta + D}) \tag{1}$$

To aid our analysis, we nondiminsionalize the system with the following substitutions:

$$x = \frac{D}{K_d} \text{ and } y = \frac{C}{K_c}.$$

These substitutions give us a system that behaves in the same manner and has the same equilibria as (1) but has fewer visible constants and simpler computations. The nondimensionalized system is

$$R_d x(1 - x) - \frac{\alpha K_c y}{\beta + K_d x}x = x\left[R_d(1 - x) - \frac{\alpha K_c y}{\beta + K_d x}\right],$$

$$(1 - y)(R_c y + \frac{\gamma \alpha K_d xy}{\beta + K_d x}) = (y)(1 - y)(R_c + \frac{\gamma \alpha K_d x}{\beta + K_d x}). \tag{2}$$

The reader must be cautious and recall while looking at our results that we will work with a dimensionless system. Results are dimensionless and should be analyzed as such.

Analysis

Equilibrium

We are interested in the long term interaction of coyotes and deer, so we look for points of equilibrium. More specifically, we are interested in whether the coyotes can control the deer, i.e. the existence of an equilibrium below the deer carrying capacity that has neither coyotes nor deer extinct. Equilibria occur at the intersections of the nullclines

$$\frac{dx}{dt} = 0 \text{ and } \frac{dy}{dt} = 0.$$

There are six intersections and thus equilibria for our system. Given as (x, y) they are

$$(0,0) \tag{3}$$

$$(0,1) \tag{4}$$

$$(1,0) \tag{5}$$

$$\left(\frac{K_d - \beta + \sqrt{(K_d - \beta)^2 - 4K_d(\frac{\alpha K_c}{R_d} - \beta)}}{2K_d}, 1\right) \tag{6}$$

$$\left(\frac{K_d - \beta - \sqrt{(K_d - \beta)^2 - 4K_d(\frac{\alpha K_c}{R_d} - \beta)}}{2K_d}, 1\right) \tag{7}$$

$$\left(-\frac{R_c \beta}{K_d(R_c + \gamma \alpha)}, \frac{R_d \beta \gamma (R_c K_d + \gamma \alpha K_d + R_d \beta)}{K_d K_c (R_c + \gamma \alpha)^2}\right). \tag{8}$$

Mathematically, coyotes controlling deer translates to an equilibrium with 0 < x < 1 and 0 < y ≤ 1. As all parameters are positive we immediately see the last equilibrium (8) with negative x value is biologically meaningless and we ignore it. It turns out that (7) does satisfy our conditions. However, to show this we must analyze the stability of the remaining relevant equilibria.

Stability

To show stability of equilibria (3) -(7), we evaluate the Jacobian at each equi-librium and examine the signs of eigenvalues. The 2 × 2 jacobian matrix is

$$\left(\begin{array}{c} R_d(1 - x) - R_d x + \frac{\alpha K_c yx K_d}{(\beta + K_d y)^2} - \frac{\alpha K_c y}{\beta + K_d x}, \\ (1 - y)(\frac{\gamma \alpha K_d y}{\beta + K_d x} - \frac{\gamma \alpha K_d^2 xy}{(\beta + K_d x)^2}), \\ -\frac{\alpha K_c x}{\beta + K_d x} \\ -R_c y - \frac{\gamma \alpha K_d^2 xy}{\beta + K_d x} + (1 - y)(R_c + \frac{\gamma \alpha K_d x}{\beta + K_d x}) \end{array}\right)$$

At (3) and (5) the eigenvalues are positive and therefore equilibrium (3) and (5) (mutual extinction and coyote extinction, respectively) are unstable.

Evaluating the Jacobian at (4), we obtain

$$\left(\begin{array}{cc} R_d - \frac{\alpha K_c}{\beta} & 0 \\ 0 & -R_c \end{array}\right)$$

As the eigenvalues of the above matrix are negative, depending upon parameter values, we see that this equilibrium is conditionally unstable. As (4) represents coyotes at carrying capacity and deer extinct, we assume this equilibrium is unstable. Thus we force one negative and one positive eigenvalue and we obtain the parameter restriction

$$R_d > \frac{\alpha K_c}{\beta}.$$

It can easily be shown that this restriction forces equilibrium (6) and (7) to have positive and negative x-values, respectively. We discard equilibrium (7), as biologically meaningless and concentrate on equilibrium (6), the only equilibrium with positive values for both x and y. We expect this equilibrium to be stable.
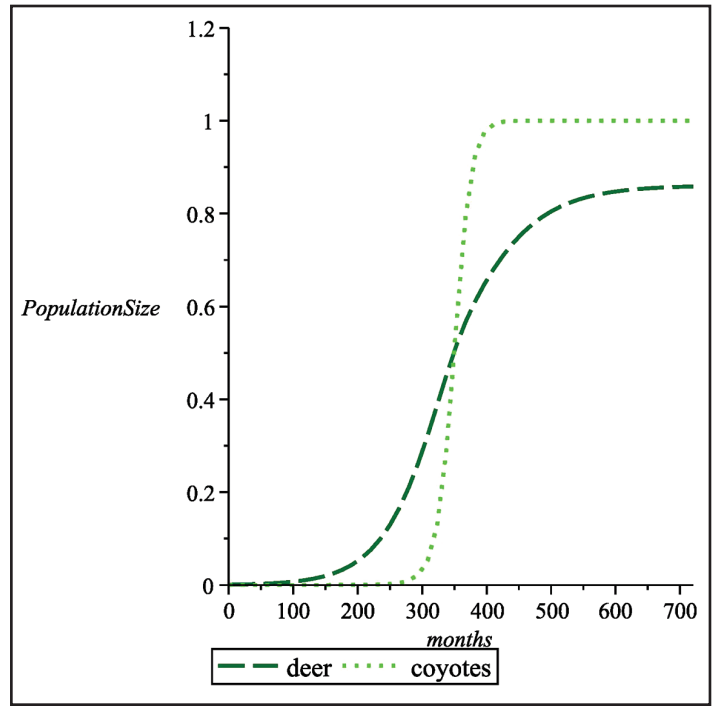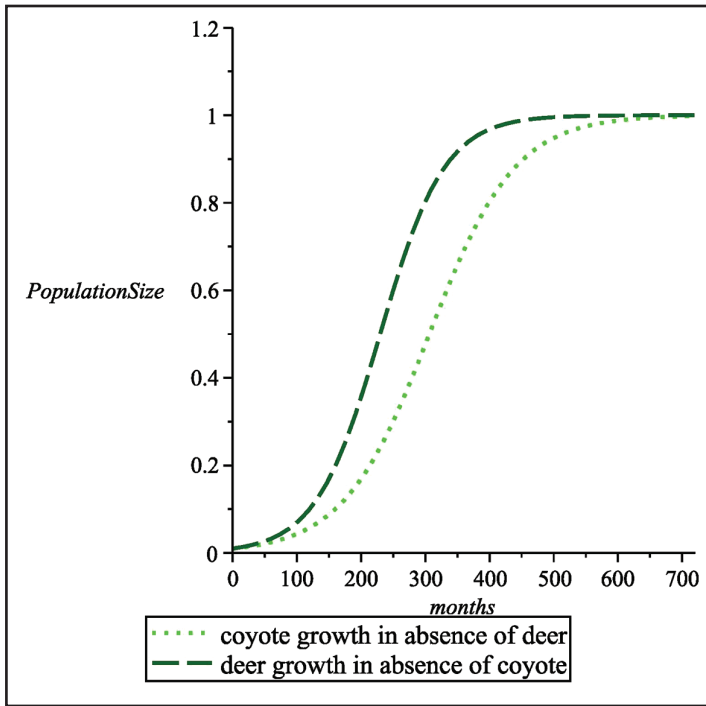
Figure 2: Time is horizontal in months. On the left, in the absence of the other population, the model shows that starting from negligible populations, deer reach high levels in 40 years and coyotes lag until they burst in 10. On the right, the system evolves with populations interacting. Note \Population Size" is scaled by the respective carrying capacity. Maximum population is 1 for graphs. (Image courtesy of Kathy Li)

Evaluating the Jacobian yields an upper triangular matrix

$$\begin{pmatrix} J_{11} & J_{12} \\ 0 & J_{22} \end{pmatrix}$$

Therefore, J_{11} and J_{22} are the eigenvalues and must both be negative for the equilibrium to be stable.

$$J_{11} = \frac{2R_d[2\alpha R_d\beta(K_c - K_d) - R_d K_d\Delta - R_d\beta\Delta + 2\alpha K_c\Delta - R_d^2\beta^2 - 2R_d^2 K_d\beta - R_d^2 K_d^2 + 4\alpha R_d K_d K_c]}{(R_d\beta + R_d K_d + \Delta)^2},$$

where

$$\Delta = \sqrt{-R_d(-R_d K_d^2 - 2R_d K_d\beta - R_d\beta^2 + 4K_d\alpha K_c)}.$$

Squaring the both sides of the above equation we obtain

$$\begin{aligned} \Delta^2 = M &= R_d^2(K_d^2 + 2K_d\beta + \beta^2 - 4K_d\alpha K_c/R_d) \\ &> R_d^2(K_d^2 + 2K_d\beta + \beta^2 - 4K_d\beta) \\ &= R_d^2(K_d - \beta)^2 > 0, \end{aligned}$$

where the rst inequality uses the restriction 9.

The above inequality is equivalent to

$$\Delta > R_d|K_d - \beta| > 0. \tag{10}$$

The three coecients of delta in the numerator of J_{11} can be written as

$$[R_d K_d + R_d\beta - 2\alpha K_c] = [R_d K_d - R_d\beta] + 2(R_d\beta - \alpha K_c) > [R_d K_d - R_d\beta]$$

Hence,

$$-\Delta[R_d K_d + R_d\beta - 2\alpha K_c] < -\Delta[R_d K_d - R_d\beta]. \tag{11}$$

Consider the six terms in the numerator of J_{11} that are independent of delta. Further, consider that due to energy transfer through tropic levels, ecosystems support more herbivores than carnivores; thus, it is reasonable that Kc < Kd. With this assumption two of the terms in the numerator are

$$2R_d(2\alpha R_d\beta(K_c - K_d)) < 0.$$

The remaining four terms can be written as

$$\begin{aligned} & 4R_d K_d K_c\alpha - 2R_d^2 K_d\beta - R_d^2\beta^2 - R_d^2 K_d^2 \\ < & 4R_d^2 K_d\beta - 2R_d^2 K_d\beta - R_d^2\beta^2 - R_d^2 K_d^2 \\ = & +2R_d^2 K_d\beta - R_d^2\beta^2 - R_d^2 K_d^2 \\ = & -R_d^2(K_d - \beta)^2. \end{aligned} \tag{12}$$

By noting that $\Delta > R_d|K_d - \beta|$, and combining (11) and (12), we obtain

$$\begin{aligned} & -\Delta[R_d K_d - R_d\beta] - R_d^2(K_d - \beta)^2 \\ = & -R_d|K_d - \beta|\{R_d K_d - R_d\beta + R_d|K_d - \beta|\} \\ = & -R_d^2|K_d - \beta|\{(K_d - \beta) + |K_d - \beta|\} < 0. \end{aligned}$$

Therefore, J_{11} is negative and one of the eigenvalues is negative. Now we check the sign of the other eigenvalue

$$J_{22} = -\frac{R_c R_d\beta + R_c R_d K_d + R_c\Delta + \gamma\alpha R_d K_d - \gamma\alpha R_d\beta + \gamma\alpha\Delta}{R_d\beta + R_d K_d + \Delta}.$$

Again as parameters are positive, the denominator is positive. Due to the negative sign in front, it only remains to show the sum of terms in the numerator of J_{22} is positive. Since there is only one negative term in the numerator, we simply show that combined with the other positive terms, the result is positive. Looking at the last three terms in the numerator and using (10) from above, we have

$$\gamma\alpha R_d K_d - \gamma\alpha R_d\beta + \gamma\alpha\Delta \geq \gamma\alpha R_d(K_d - \beta) + \gamma\alpha R_d|K_d - \beta|$$

As the last three terms in the numerator of J_{22} are non-negative, the entire numerator is positive and J_{22} is negative.
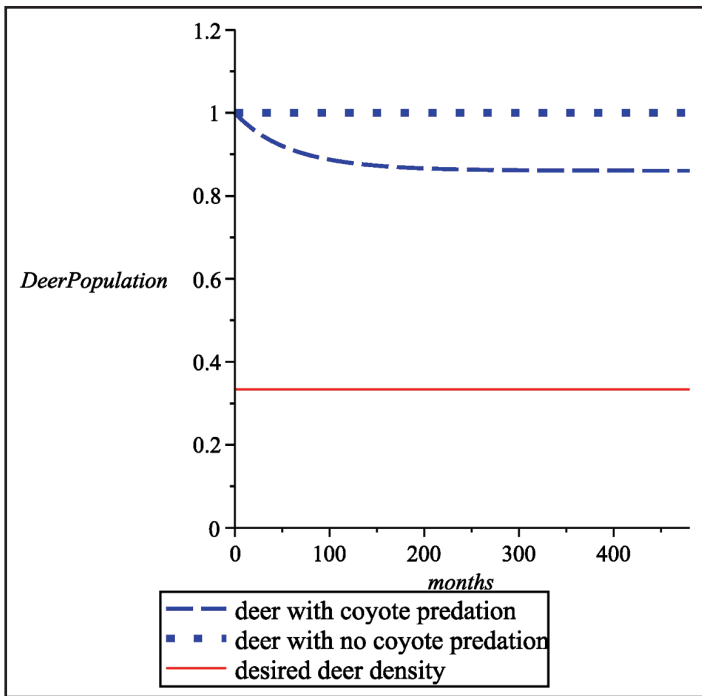
Figure 3: We start our model with coyote and deer populations at their respective carrying capacities, their current levels. We see the deer stay at their carrying capacity Kd indenitely without predation but fall nearly 20 % to a new equilibrium (yet still above desired levels) when predation is considered. (Image courtesy of Kathy Li)
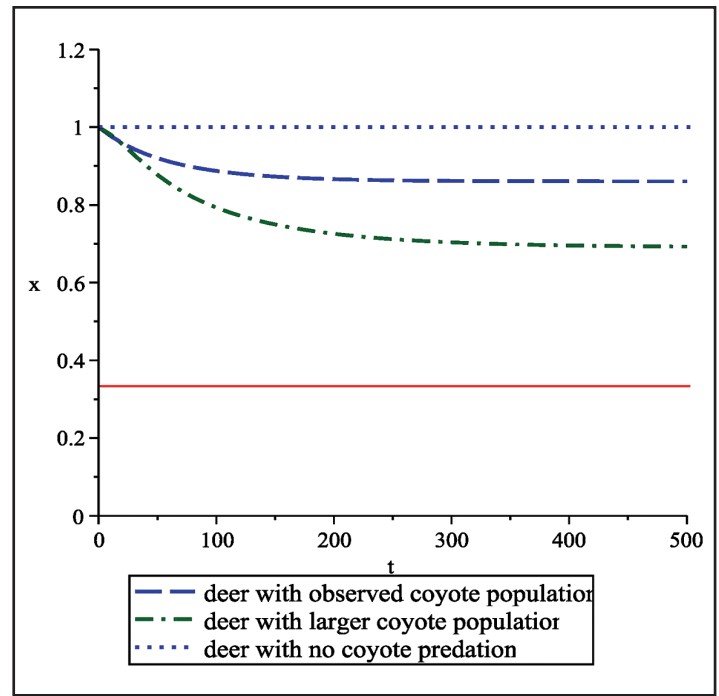

Figure 4: The graph above depicts the eect of setting Kd=20, the theoretical healthy deer carrying capacity. At this level, the model incorrectly predicts deer will quickly decrease and the population will self-correct to heathy levels without intervention in short time. Dense deer populations do not seem to be naturally decreasing so we must set Kd=60, the current population density to get realistic behavior for the model (Figure 3 has Kd=60). (Image courtesy of Kathy Li)

Figure 5: Increasing Kc, the maximum coyotes allowed by humans in the area, increases predation and control on deer but still cannot bring deer to desired levels.(Image courtesy of Kathy Li)
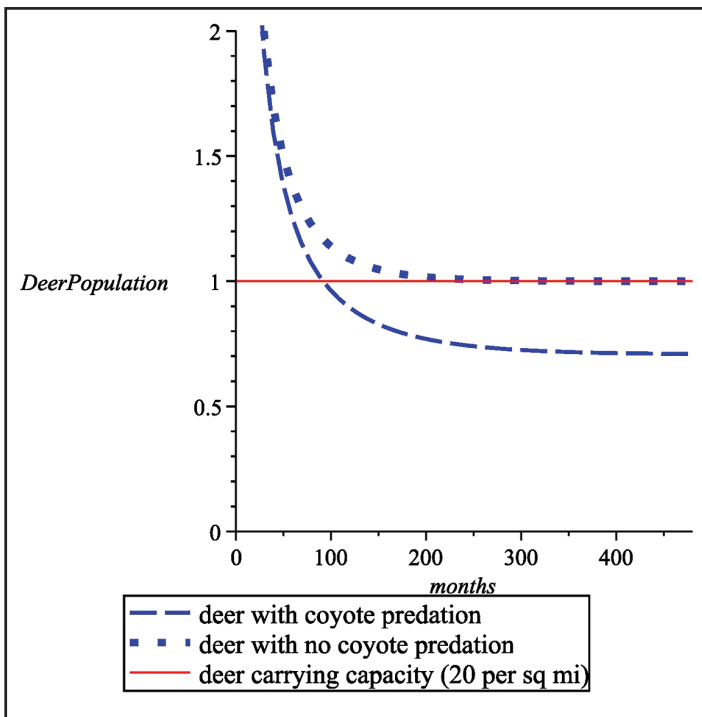
In conclusion, both eigenvalues J_{11} and J_{22} are negative and equilibrium (6) is stable. This means that our coyote-deer system has a stable equilibrium below the maximum number of deer and thus predicts that coyotes will have some controlling effect. Now we numerically examine the extent of this effect.

Numerical Investigation

Setting Parameters

We consider ranges of the parameters which have been gathered from the liter-ature and refine these ranges by choosing values which best model the historical growth of the two populations. The values

$$R_d = .02, R_c = .015, K_d = 60, K_c = .2, \gamma = .1, \alpha = 1, \beta = 20$$

gave us results which best correlated with the historical dynamics described: deer population suffered until the 1950's but exploded by the 1990's [22] and coyote observations were sparse in the late 1950's but more common around the 1970's [6, 19]. This can be seen in Figure (2).

Model Results

We start the model with deer and coyotes at their respective carrying capacities, as these are the presumed current levels, and use MapleTMsoftware to numeri¬cally solve and plot our system. We consider evolution of the system over the next
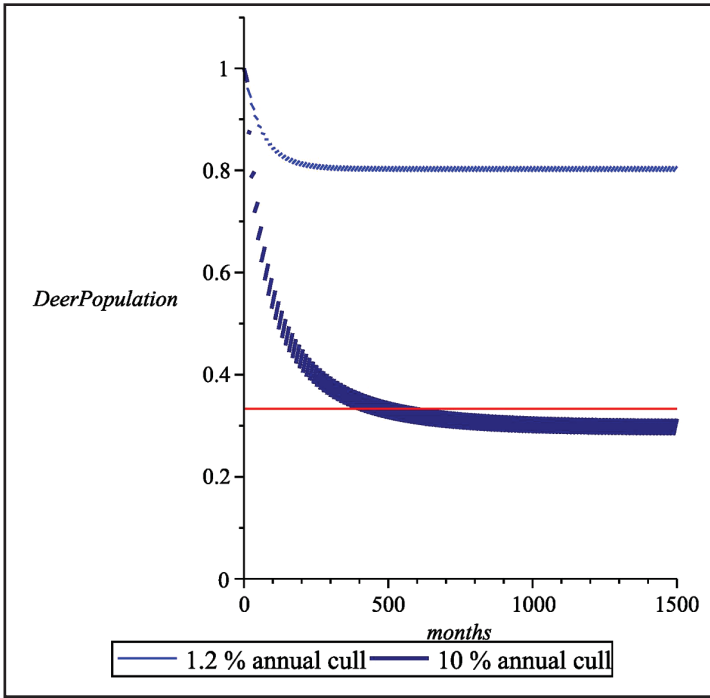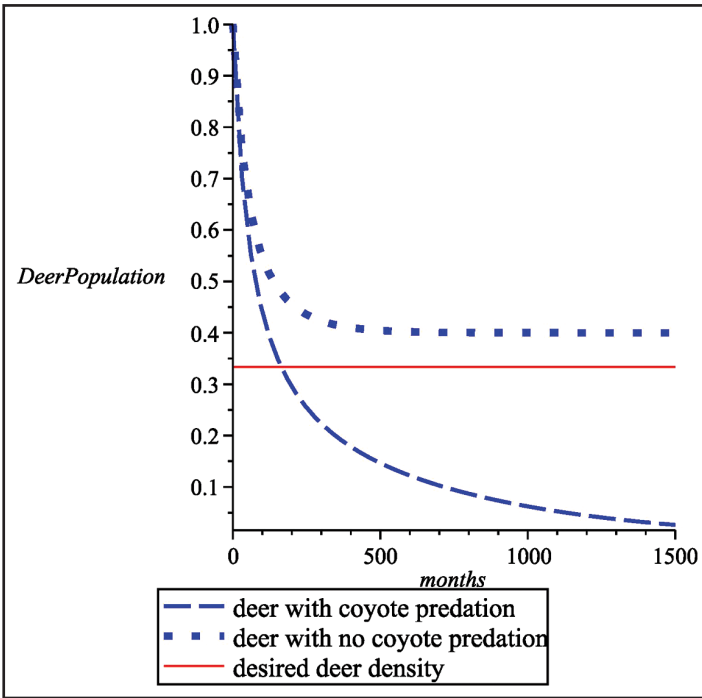
Figure 6: The left graph shows 1.2% continuous culling and the importance of considering coyote predation in the culling strategy. The graph on the right considers a discrete (seasonal) cull and shows a large dierence from continuous culling model predictions. (Image Courtesy of Kathy Li)

fifty years in Figure 3. We see that without predation, deer stay at their carrying capacity but with coyote predation, deer population drop to the level of (6), the previously found equilibrium. This shows us that coyotes do and will have an effect on deer populations.

The thin horizontal line in Figure 3 represents 20 deer per square mile which is a third of the current deer population levels and is thought to be a natural, healthy level that deer existed at in the presence of the wolf and that deer can continue to exist at without overpopulating the area [15, 4]. This is a level that is recommended by state officials for deer control programs [22, 11]. To begin to control Lyme Disease, municipalities would even like to see deer below this recommended level [5].

Deer Carrying Capactity K_d

As previously discussed, we take carrying capacity to be constant. In Figure 3 Kd is set to current level of deer populations, 60 deer per square mile. However, we can also set Kd to what the literature suggests is a healthy environ¬tal carrying capacity, 20 deer per square mile. At this level, the logistic term causes the deer population to naturally fall to the healthy environmental carrying capacity of 20 deer per square mile within 300 months without any human intervention or predation. Even with an artificially high initial condition of 10 times the natural carrying capacity or 200 deer per square mile and no coyote predation, the deer fall to reasonable densities fairly quickly as see in Figure 4. This does not reflect reality as deer populations have

remained well above the healthy carrying capacity of 20 deer per square mile and have not fallen nor are they showing signs of steep decline. Thus clearly, we cannot use the theoretical carrying capacity but must use the current population levels and let Kd=60.

Additional Coyotes

It is evident from Figure 3 that at their current densities of .2 coyotes per square mile, coyotes will not be able to control deer population to the desired levels. If we are correct in assuming that humans control coyote population size to a certain density Kc, then we can consider what would happen if humans would allow more coyotes to inhabit the area. Perhaps, if coyotes were fostered in the area to act as biological control on deer, we would see reduced numbers of deer.

We study this scenario by raising the value of Kc from .2 to .38, a doubling of the population from current levels in Massachusetts [27]. Figure 5 shows that even doubling the coyote population, while having a larger effect on the deer, will not effectively get deer to or maintain deer at desired levels.

We see that on their own, and at higher levels, coyotes will not control deer population. Even with larger populations (which are unrealistic due to the dense population in the area who are already reluctant to share the landscape with the carnivore), coyotes cannot control the deer. Hence, we now consider other efforts that would have to be taken to control deer and add the effect of culling to our model.
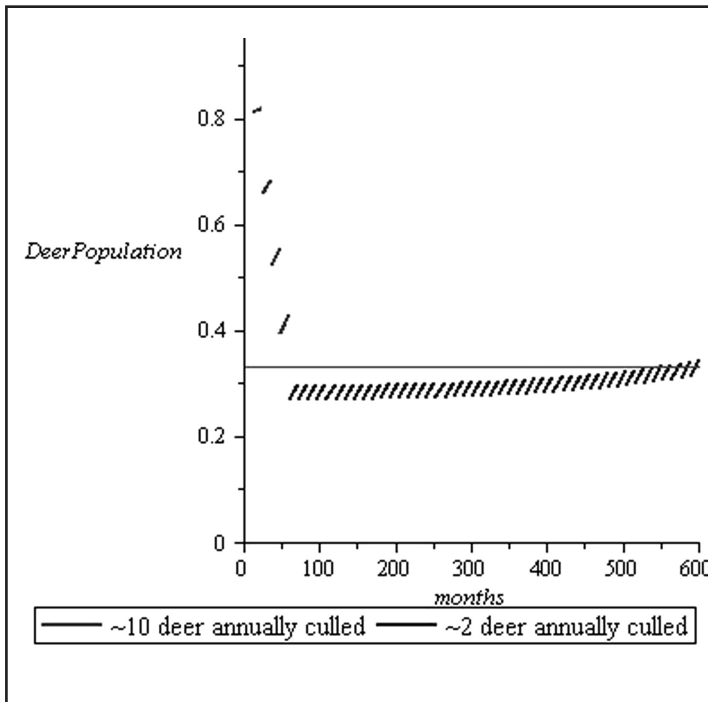
Figure 7: Proposed plan for effective culling strategy with coyote predation. Cull at high rates for 5 years and then lower rates to maintain over 50 year period. (Image courtesy of Kathy Li)

Culling

Analytically, we found that coyotes would indeed control the deer population to some extent. The level that coyotes could keep the deer at are below the observed densities of 60 deer per square mile as illustrated in Figure 4. However, within the range of realistic parameters, our model does not show that coyotes are capable of controlling deer to or below the 10 deer per square mile densities that are presumed optimal for controlling Lyme disease or reducing the nuisance of deer. Thus, we establish that external forces, such as active culling, will be needed to control deer and we look at some of the types of culling.

We begin our study by reconsidering deer growth and modifying our original equation to take into account a loss of deer due to an external active cull. We consider two types of culling: proportional and lump.

We consider the culling strategy that takes a proportion of the population by introducing μ, the proportion of deer killed. Frequently this sort of culling is used for modeling, as it allows for simpler analysis than the more realistic constant value or lump culling.

Continuous Proportional Culling

We consider the culling strategy that takes a proportion of the population by introducing μ, the proportion of deer killed. Frequently this sort of culling is used for modeling, as it allows for simpler analysis than the more realistic constant value or lump culling. The dimensionless culling model is

$$(a) \quad \frac{dx}{dt} = R_d x(1-x) - \frac{\alpha K_c y}{\beta + K_d x}x \quad = x\left[R_d(1-x) - \frac{\alpha K_c y}{\beta + K_d x}\right] - \mu x,$$

$$(b) \quad \frac{dy}{dt} = (1-y)(R_c y + \frac{\gamma \alpha K_d xy}{\beta + K_d x}) \quad = (y)(1-y)(R_c + \frac{\gamma \alpha K_d x}{\beta + K_d x}).$$

Figure 6 shows the sensitivity of the model to coyote predation. Killing only 1.2% of the deer population has a drastic effect of reducing deer. However this model also illuminates the importance of coyotes. With the increased pressure of culling, deer are more susceptible to the effect of coyote predation.

However this model seems unrealistic as culling 1.2% of deer is an extremely small portion of the deer. It is projected that much larger culls will be needed to control deer populations. We propose that the fallacy of this model is due to it assuming that the culling is continuous. This means the model is continuously taking 1.2% of the deer which is clearly unrealistic. Culling programs tend to be administered through seasonal culls or taking deer at only one period in the year and not continuously. We would like to take this seasonal culling into consideration, and thus we need a way to make the effect of culling on our model discrete.

Discrete (Seasonal) Culling

To study a realistic seasonal, proportional culling model, we utilize numerical approximating packages in MapleTM . We write a simple program which runs the system for a year, stops and subtracts a proportion μ of the deer population, and then uses that value as an initial condition for the next year. This program outputs a plot of projected yearly deer population values.

We see that with discrete culling much larger proportions of deer need to be killed in order to control populations (Figure 6). This is the more realistic result that we hoped to achieve with our MapleTMprogram. However one problem remains: time. Looking at the horizontal axis we see that culling a set proportion takes too long to get deer to or below desired levels.

Conclusion

We looked at the potential of the eastern coyote as a natural biological control on deer populations. Analytically, we found one interesting stable equilibrium in which the coyotes control the deer. Numerically analyzing this equilibrium for reasonable parameters, we find that although the coyote do have some effect on controlling deer populations, it is not enough to keep deer populations below the desired levels (20 deer per square mile). Therefore, we investigated potential anthropogenic control through culling strategies. We found that modeling continuous culling was unrealistic so we wrote a program to represent seasonal culling. This annual culling model gave far more realistic predictions yet still showed that it would take too long to control the deer if the proportion culled each year was kept constant. In conclusion, we proposed the following plan: Cull deer at high rates for a short time period, say five years, and then at more modest rates to maintain healthy populations. The predictions for such a

strategy can be see in Figure 7.

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